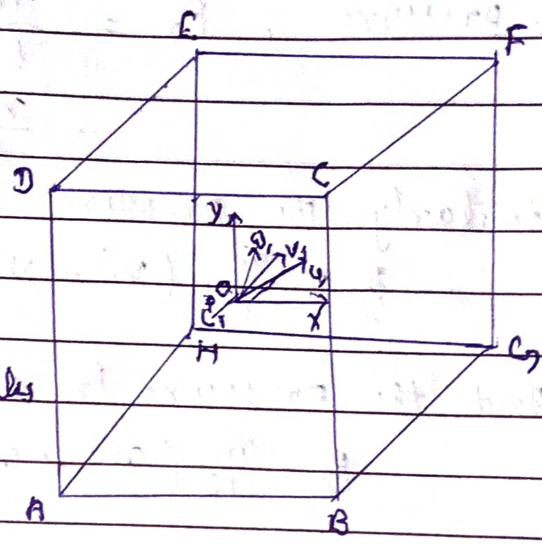
* Expression for the Pressure of a Gas :—

Let us consider cubical vessel ABCDEFGH of side l cm containing the gas. The volume of vessel and hence that of the gas is l^3 cc. Let n and m represent the very large no. of molecules present in the vessel and the mass of each molecule respectively.



Let us consider P moving in a random direction with a velocity C_1. The velocity can be resolved into three perpendicular components u_1, v_1 and w_1 along the x, y and z axis respectively. Therefore,

$$c_1^2 = u_1^2 + v_1^2 + w_1^2$$

The component of the velocity with which the molecule P will strike the opposite face BCFG is u_1 and the momentum of molecule is mu_1. This molecule is reflected back with the same momentum mu_1 in an opposite direction and after travelling a distance l will strike the opposite face ADEH.

The change in momomentum produced to the impact is

$$mu_1 - (-mu_1) = mu_1 + mu_1 = 2mu_1$$

As the velocity of molecule is u_1, the time interval between two successive impacts on the wall BCFG is $\frac{2l_1}{u_1}$ seconds

$\therefore$ No. of impacts per second $= \frac{1}{2l/u_1} = \frac{u_1}{2l}$

Change in momentum produced in one second due to the impact of this molecule is

$$2mu_1 \times \frac{u_1}{2l} = \frac{mu_1^2}{l}$$

The force Fx due to the impact of all the n molecules in one second $= \frac{m}{l}\left[u_1^2 + u_2^2 + \cdots + u_n^2\right]$

Force per unit area on the wall BCFG or ADEH is equal to the pressure

$$P_x = \frac{m}{l \times l^2}\left(u_1^2 + u_2^2 + \cdots + u_n^2\right)$$

$$= \frac{m}{l^3}\left(u_1^2 + u_2^2 + \cdots + u_n^2\right)$$

Similarly, the pressure P_y on the walls CDEF and ABGH is

$$P_y = \frac{m}{l^3}\left(v_1^2 + v_2^2 + \cdots + v_n^2\right)$$

And the pressure P_z on the walls ABCD and EFGH is

$$P_z = \frac{m}{l^3}\left(\omega_1^2 + \omega_2^2 + \cdots \omega_n^2\right)$$

As the pressure of a gas is the same in all directions the mean pressure P is given by

$$P = \frac{P_x + P_y + P_z}{3}$$

$$= \frac{m}{3l^3}\left[(u_1^2+v_1^2+\omega_1^2)+(u_2^2+v_2^2+\omega_2^2) + \cdots + (u_n^2+v_n^2+\omega_n^2)\right]$$

$$= \frac{m}{3l^3}\left[C_1^2 + C_2^2 + \cdots + C_n^2\right] \quad \text{— (i)}$$

But Volume $V = l^3$.

Let C be the root mean square velocity of molecules

Then, $$C^2 = \frac{C_1^2 + C_2^2 + \cdots + C_n^2}{n}$$

$$\therefore nC^2 = C_1^2 + C_2^2 + \cdots + C_n^2$$

Putting this value in eqn (i), we get

$$P = \frac{m.nC^2}{3V} \quad \text{— (ii)}$$

But $M = m.n$ where M is the mass of Gas of volume V, m is the mass of each molecule and n is the no of molecules in volume V

$$\therefore P = \frac{MC^2}{3V}$$

$$\therefore P = \frac{1}{3}\rho C^2 \quad \text{— (iii)}$$

$$\because \frac{M}{V} = \rho = \text{the density of the gas.}$$

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From eqn (iii)

$$C^2 = \frac{3P}{\rho}$$

$$\therefore C = \sqrt{\frac{3P}{\rho}} \quad \text{— (iv)}$$

Note :- R.M.S. velocity C is the square root of the mean of squares of the individual velocities and it is not equal to the mean velocity of the molecules.